Model Predictive Control: A case study of Trøjborg

Modèle de contrôle prédictif: une étude de cas de Trøjborg

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RÉSUMÉ

La survenue de plus fortes précipitations au cours des dernières années a mis davantage l'accent sur les moyens de prévenir les débordements des réseaux d'égouts, en particulier les méthodes visant à améliorer l'utilisation des infrastructures actuelles. Une méthode consiste à améliorer le contrôle du système. Dans cette étude, nous nous intéressons à la méthode du Modèle de contrôle prédictif (MPC) appliquée à une étude de cas d'un réseau d'égout réel. La force du MPC réside dans sa capacité à agir de manière préventive selon les prévisions météorologiques, tout en tenant compte des contraintes. L'étude de cas porte sur le bassin versant de Trøjborg dans la ville d'Aarhus, au Danemark. Il est utilisé pour comparer la méthode du MPC à la méthode de contrôle actuelle, avec pour objectif principal la réduction du volume de débordement. La méthode de contrôle actuelle est celle du contrôle basé sur des règles, qui fonctionne rétroactivement sur le système, et dont l'efficacité dépend de la qualité et de la quantité des règles. La comparaison se fait à partir de simulations réalisées avec le logiciel haute fidélité MIKE URBAN. En conclusion, l'utilisation de la méthode du MPC pour les réseaux d'égouts fait apparaître une nette amélioration.

ABSTRACT

Occurrence of larger rain events in recent years has increased the focus on how to prevent overflows from sewer systems, especially on methods for improving utilization of current infrastructures. One method is improving the control of the system. In this study, we consider the control method Model Predictive Control (MPC) applied in a case study of a real sewer system. The strength of MPC is the ability to pre-emptive act on future weather prognosis, while handling constraints. The case study is the Trøjborg catchment in the city of Aarhus, Denmark. It is utilized to compare the MPC method and the current control method, with focus on reducing overflow volume. The current control method is rule-based control, which works retroactively on the system. With efficiency reliant on the quality and quantity of the rules. The comparison is based on simulations with the high-fidelity software MIKE URBAN. In conclusion, the utilization of the MPC method in sewer systems shows a significant improvement.

KEYWORDS

MPC, Overflow, Predictive Control, Urban drainage, Water Smart Cities

1 INTRODUCTION

With the expected increases in heavy rain events in Denmark (Gregersen 2015), the capacity of the sewer system becomes important with regard to preventing floods and weir overflow. In order to increase the capacity of a sewer system, it requires an expansion of the existing infrastructure of the sewers, which is an expensive and time costly procedure. An alternate approach is to increase the efficiency of the utilization of the existing infrastructure; this can be done by utilizing control methods to optimize the utility of the volume capacity.

In this work, we consider the control method "Model Predictive Control" (MPC), previously suggested for sewer systems[2]. The benefits of the MPC method is the ability to take future rain predictions in consideration, while finding the optimal solution based on a given set of criteria, which current controllers cannot. Furthermore, the MPC method is capable of handling the constraints of the system, both physical limitation as well as operational desired constraints. For demonstration of the utility of the MPC method, we consider a case study of the catchment of the Trøjborg area in the city of Aarhus in Denmark. The catchment has been modelled with the high-fidelity software MIKE URBAN (MIKE 2018), and will be utilized as a substitute for real testing for practical reasons.

Fig. 1: Trøjborg Catchment with detention tank at the blue circle, yellow main pipes and purple connection to neighbouring catchment

2 CASE AND METHOD

The geographical catchment area of the case study, Trøjborg, can be seen in fig. 1.

2.1 Catchment

The Trøjborg catchment is a simple catchment covering a geographical area of approximate 5 km^2. The catchment has only downstream connections to other catchments and it contains a single detention tank. The detention tank is connected to the catchment in two ways; by internal overflows from two orifices and by passive inflows directly from the catchment

The current controller at the Trøjborg catchment is a rule-based controller (Garcia et al. 2015), with the objective to empty the tank whenever it is possible. For this reason, it is equipped with a sensor for measuring the outflow of the catchment. The rule-based controller utilizes the measurement to manipulate the outflow, by controlling two orifices that can divide the catchment inflow towards the detention tank and a pump used for emptying the detention tank. MIKE URBAN's representation of the rule-based controller is used as the baseline controller for comparison of the two control methods. For the MPC, the MIKE URBAN model has been modified slightly, such that the pump has its own flow sensor placed directly after the pump downstream. While the orifices were set to be static and open (i.e. maximum bypass of the detention tank).

2.2 Model Predictive Control

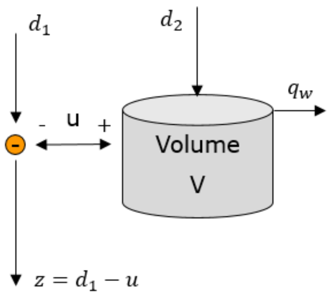

Fig. 2: Single State Model

When utilizing MPC (Gelormino & Ricker 1994), one chooses a discrete design model to describe the system of interest. In this work, a simple single state model was chosen for the controller, seen in fig. 2. The model inflows d_1 and d_2 are the flows from the catchment above the detention tank. The model output z is outflow of the catchment, given as the difference between the inflow d_1 and control flow u. The state of interest is the volume of the detention tank, being dependent on the inflow d_2, the overflow q_w and the flow u. The control flow u is a collection of flows to and from the detention tank: the overflows from the orifices (the junction in fig. 2), as well as the pump flow to the junction. For this MPC, the flow u is considered the control variable. The MPC method utilized in this work is of the quadratic type, meaning it is defined as a quadratic program, and therefore requires the design model of the sewer system to be linear, as seen in eq. 1 and 2. For this reason, the overflow of the detention tank is excluded from the linear model.

$$V(k+1) = V(k) + \Delta T\, u(k) + \Delta T\, d_2(k) \quad (1)$$

$$z(k) = d(k) - u(k) \quad (2)$$

Where at sample *k*, $V(k)$ is the internal state of the system corresponding to the volume, the uncontrolled variables $d_1(k)$ and $d_2(k)$ are the two catchment inflows, and the output $z(k)$ is the outflow of the catchment. The excluded overflow q_w is in this MPC handled as a soft constraint on the maximum volume, allowing the volume to exceed its maximum by a volume s. The parameter ΔT is the sampling time of the discrete model, such that time $t = k\Delta T$. By constructing a cost function J, the desired objective of the controller can be defined. For a quadratic MPC, the cost function contains quadratic and linear terms, and the constraints are linear, as seen in eq. 3-7.

$$J = \min_{z,\Delta u,s} \frac{1}{2}\left(z - z_{ref}\right)^T Q\left(z - z_{ref}\right) + \frac{1}{2}\Delta u^T R \Delta u + \frac{1}{2} s^T R_s s \tag{3}$$

$$x_{min} \leq x(k) \leq x_{max} + s(k) \tag{4}$$

$$z_{min} \leq z(k) \leq z_{max} \tag{5}$$

$$u_{min} \leq u(k) \leq u_{max} \tag{6}$$

$$\Delta u(k) = u(k) - u(k-1) \tag{7}$$

$$0 \leq \mathrm{s(k)} \tag{8}$$

In this work, the cost function consists of the following quadratic objectives; I) minimization of the deviation from the outflow reference II) minimization of the change of control flow III) minimization of the overflow volume *s*. The variable Δu is the change of control flow u. The matrices Q, R and R_s are the weights of each of the objectives, here chosen as uniform diagonal matrices. With the relative weighting being $R_s > Q > R$, indicating a larger penalty on overflows than on outflow deviation and control flow change.

3 RESULTS

The evaluation of the MPC controller is possible by utilizing the high fidelity simulation program MIKE URBAN with a model of the Trøjborg catchment. For the evaluation, the catchment is loaded with two types of rain events: scaled historical rain and theoretical blocks of rain. The rain events of both types are chosen, so that they have different durations and rain intensities. It is assumed the rain intensity is uniformly distributed over the entire catchment. In the results shown, rain events for which simulations did not produce any overflow in neither the MPC nor the Baseline are excluded.

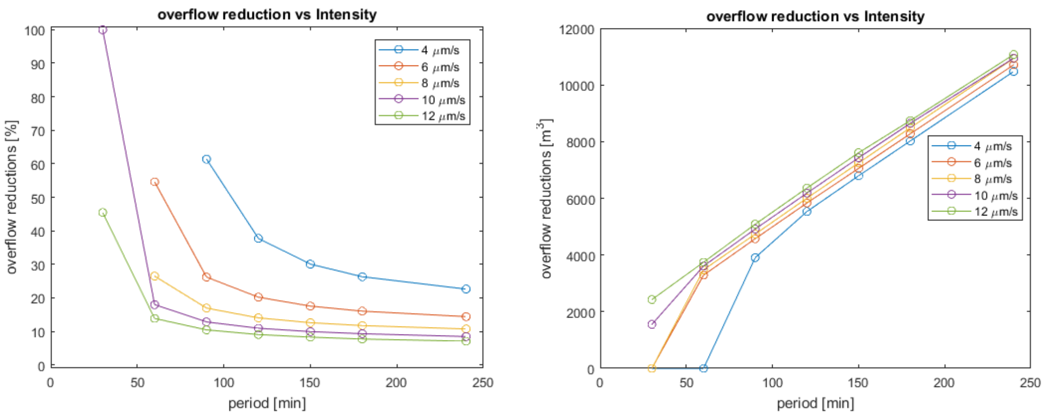

Fig. 3: percentage $\left[1 - \frac{V_{cso}^{mpc}}{V_{cso}^{base}}\right]$ *and volumetric* $\left[V_{cso}^{base} - V_{cso}^{mpc}\right]$ *overflow reduction of block rain*

For the simulations with the block rain, the period varies from 30 min to 240 min in steps of 30 min or 60 min. The rain intensity varies from 4 µm/s to 12 µm/s. For comparison, in Denmark a cloudburst has a rain intensity of at least 8.3 µm/s corresponding to minimum 15 mm over a period of maximum 30 min (Nielsen et al. 2017).

From fig. 3, it can be observed that the efficiency of the MPC controller in comparison to the baseline controller appears to converge as the period of the rain block increases. The percentage improvement in the left panel appears to converge downwards to approximate 10% improvement for the heavier block rains. The absolute overflow volume reduction, shown in the right panel, appears to

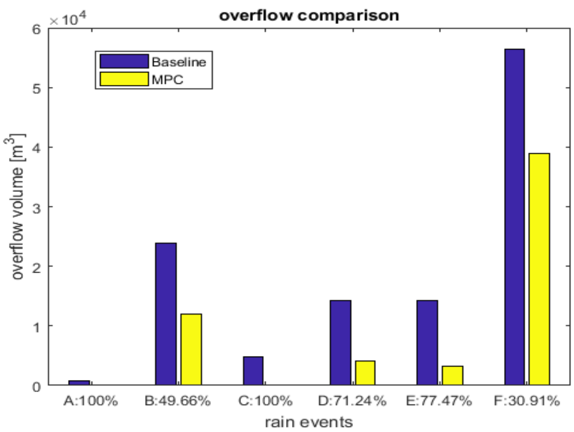

Fig. 4: weir overflow volume comparison of simulations with historical data

converge to a linear relation with the period, with the rain intensity variating the slope and offset slightly.

The historical rain events utilized, are based on mean area rainfall for the Trøjborg catchment extracted from the utility company's operational system. The overflow from six rain events are shown in fig. 4, it can be seen that the MPC controller provides less overflow volume than the baseline controller, and in two cases(A and C) the overflow is eliminated. In the figure, each event is listed with the corresponding percentage reduction shown at the base, and it can be observed that the percentage improvement decreases with the increase of the absolute overflow volume. The results with scaled historical rain scenarios confirm the results of the simple block rain scenario, that the MPC notably reduces the overflow volume generated during the rain events. In fig. 5-6, the overflows from the events B and D are shown. The overflow is not only reduced in amplitude, but also occurred for shorter periods in comparison to the baseline controller.

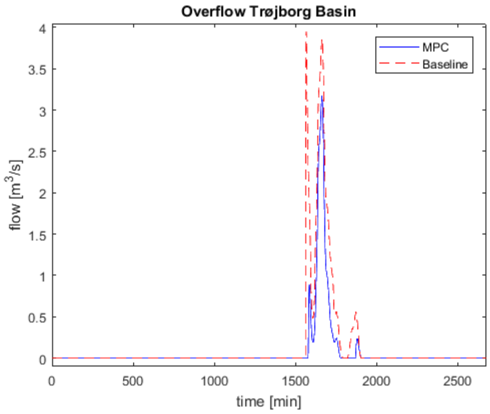

Fig. 5: overflows for event B

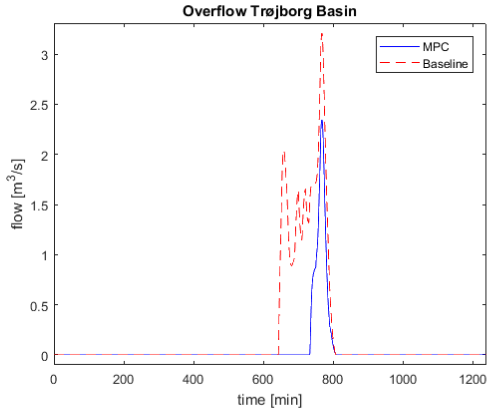

Fig. 6: overflows for event D

4 CONCLUSION / OUTLINE

This study probes the utility of the MPC method with regard to sewer systems, in comparison to the current control method, through a case study of the Trøjborg catchment in Denmark, by simulations with MIKE URBAN. The results of the simulations show a clear benefit of the MPC controller over the baseline's rule-based controller, with the generated overflows being reduced significantly. The converging linear relation found between the reduction in overflow volume and the rain period, indicates that the reduction volume per time interval has reached a physical limit due to the outflow pipe being full.

ACKNOWLEDGEMENT

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